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Resummations, Power Corrections and Interjet Radiation[1]

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Abstract

Resummation in QCD provides insight into the evolution of final state jets from short to long distances, and of accompanying interjet radiation. Applications to event shapes, including the recently-proposed angularities, suggest experimental tests of the interrelations between weak- and strong-coupling dynamics.

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1 Introduction

Jets are the imprint on final states of dynamics at short distances, whether from transitions within the standard model (Drell-Yan annihilation or QCD scattering, for example) or from the creation of new degrees of freedom. An individual jet, however, provides scant clues to its origin. Indeed, a jet is simply a collection of co-moving particles in relative isolation. The number of particles in a given jet, and consequently its total energy and momentum, are never uniquely defined. Nevertheless, the distributions of particles and momenta within and between high energy jets encode information at all scales, from the largest momentum transfer or mass, through QCD evolution to the scale of the strong coupling, $\Lambda_{\rm QCD}$. From the point of view of quantum field theory, the system passes through a stage of weak coupling at the shortest distances on to strong coupling and nonperturbative dynamics at the longest. Starting with perturbation theory, resummations provide one bridge between these regimes.

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2 Resummation: Why? When? How?

Only the most inclusive observables depend on a single hard scale. More common – and more interesting – are functions of two ordered but perturbative scales, $Q \gg Q_1 \gg \Lambda_{\rm QCD}$. Varying the lower scale, Q_1 allows us to move continuously between the ultraviolet and the infrared. Typically, as Q_1 decreases, the perturbative series develops one or two logarithmic enhancements in the ratio Q/Q_1 for every power of α_s. These enhancements may often be organized, that is resummed, to all orders in perturbation theory. Examples include enhancements that are explicit in the perturbative cross section, such as those in the transverse momentum distributions ($Q_1 = Q_T$) of vector bosons [1], and enhancements that are implicit in inclusive distributions, as in threshold resummation for inclusive Higgs production [2], integrated over Q_T.

Perturbative calculations in QCD depend on the fundamental property of asymptotic freedom. The most tractable perturbative quantities are one-scale cross sections that are fully infrared safe, that is, cross sections that can be expressed as series in α_s with finite coefficients. Infrared safe observables are essentially descriptions of the flow of energy [3]. The class of infrared safe, single-scale cross sections is limited to e^+e^- total and jet cross sections, however, and the latter are single-scale only when the masses of the jets are comparable to the total energy Q. When jet masses become small compared to Q, it is necessary to resum, even in these infrared safe cross sections.

Resummation of two-scale logarithms can be derived whenever a cross section (sometimes amplitude) is a product or convolution of factors that separate the disparate scales Q and Q_1 through the introduction of a third scale, the factorization scale, $\mu \gg \Lambda_{QCD}$. Schematically, a factorizable cross section σ that depends on a very large momentum scale Q and a much softer scale m can be written as $\sigma(Q, m) = \omega(Q, \mu) \otimes f(\mu, , m)$. Here m labels the soft scale(s), typically light quark masses and $\Lambda_{\rm QCD}$, although it may also represent the lower of two perturbative scales (say a jet mass) in an infrared safe cross section. Whenever there is such a factorization, there is evolution. Since the physical cross section cannot depend on the factorization scale, the variation of the short distance function, ω, with μ must cancel that of the long distance function, f,

$$\mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m) = 0 \quad \Rightarrow \quad \mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}\,, \tag{1}$$

where the "kernel" P can depend only on the variables that the two functions hold in common. Wherever there is evolution there is resummation [4], which is simply the solution to the evolution equation or equations. An alternative route to resummed cross sections is based on coherent branching, which analyzes repeated gluon emissions [5]. One may think of each branching as a "minifactorization", an incremental step from short to long distances.

Factorization proofs [6] are based on the quantum incoherence of dynamics at incommensurate length scales, and of the evolution of systems of particles mutually receding at nearly the speed of light. These considerations are reflected in the structure of factorizable cross sections near an "elastic" limit, with final states that may be characterized by a single hard scattering and a fixed

number of jets,

$$\sigma(Q, a+b \to N_{\rm jets}) = H \otimes \mathcal{P}_{a'/a} \otimes \mathcal{P}_{b'/b} \otimes \prod_{i=1}^{N_{\rm jets}} \; J_i \; \otimes S\,. \tag{2}$$

The convolutions may be in partonic momentum fractions, transverse momenta or energies, depending on the observable. Physics at the hard scale is in H, while the remaining functions generate perturbative logarithms and nonperturbative dependence. In effect, the calculation of any such cross section can be broken down into a set of standard components: the product of $\mathcal{P}$'s, which describe how two partons a' and b', which result from the quantum evolution of partons a and b, collide at H, leaving behind forward jets of "spectators", and producing a set of outgoing jets, J_i and coherent soft emission S. For example, in hadronic collisions when partonic energy is just sufficient for W or Z production, the cross section takes the form $H \otimes \mathcal{P}_{q/a} \otimes \mathcal{P}_{\bar{q}/b} \otimes S$. Such an inclusive cross section is sensitive to long distance dynamics only through the forward jets and the soft radiation. Similarly, for $\mathrm{e}^+\mathrm{e}^- \to 2\mathrm{J}$ the cross section can be factorized into a hard part times two jets, $H \otimes \; J_q \otimes J_{\bar{q}} \otimes S$, while a DIS structure function near $x = 1$ breaks up into $H \otimes \mathcal{P}_{q/a} \otimes J_q \otimes S$. Because the cross sections factorize in these limits, their logarithmic dependence on the masses of the jets and on $1 - x$, respectively, may be resummed [4, 5, 7].

3 Application: Angularities in $\mathrm{e}^+\mathrm{e}^-$

Among the many applications of resummation, event shapes near the two-jet limit in $\mathrm{e}^+\mathrm{e}^-$ annihilation have received perhaps the most attention, in large part because of the large lever arm provided by the LEP data, both in the large scale, the total c.m. energy, and in the smaller scale, typically the jet mass.

A generalized class of event shapes, the "angularities", were proposed in Ref. [8] (reanalyzed and renamed by Berger and Magnea in [9]), with the motivation of providing a new parameter that interpolates between distinct traditional measures of jet substructure. For final state N, we define

$$\tau_a = \frac{1}{Q} \sum_{i\,{\rm in}\,N} E_i \; (\sin\theta_i)^a \; (1 - |\cos\theta_i|)^{1-a} \equiv \frac{1}{Q} \sum_{i\,{\rm in}\,N} E_i \, w_a(\cos\theta_i)\,. \tag{3}$$

Here θ_i is the angle between the direction of final-state particle i and the thrust axis. Special cases are $a = 0$, the thrust, $a = 1$ broadening, and the total cross section, $a \to -\infty$. The "elastic" limit for any finite a is $\tau_a = 0$, where the final state consists of two perfectly collimated jets. Following the reasoning outlined above to next-to-leading logarithm (NLL) in τ_a, the differential cross section factorizes in the space of Laplace moments [8],

$$\begin{aligned} \sigma\left(\tau_a, Q, a\right) &= \sigma_{\rm tot} \int_C d\nu \, \mathrm{e}^{\nu\,\tau_a} \; \left[\, J_i(\nu, p_{Ji})\,\right]^2 \,, \\ J_i(\nu, p_{Ji}) &= \int_0 d\tau_a \, \mathrm{e}^{-\nu\tau_{Ji}} \; J_i(\tau_{Ji}, p_{Ji}) = \mathrm{e}^{\frac{1}{2}E(\nu,Q,a)}\,, \end{aligned} \tag{4}$$

into a product of jet functions, J. At the level of NLL, it is possible to *define* $S_{c\bar{c}} = 1$, which essentially serves as a definition of the jets. This definition is equivalent to the calculation of the cross section in terms of jets evolving independently according to coherent branching [5].

Logarithms in the transform variable ν are in close correspondence with those in τ_a. In transform space when $a < 1$, logs of ν exponentiate in Sudakov form, with up to $n+1$ logarithms at order α_s^n in the exponent E in Eq. (4), given in terms of known anomalous dimensions $A(\alpha_s)$ and $B(\alpha_s)$ by

$$
\begin{aligned}
E(\nu, Q) \; &= \; 2\int_0^1 \frac{du}{u}\Bigg[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a} - 1\right) \\
&\quad + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right)\left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1\right)\Bigg]. \qquad (5)
\end{aligned}
$$

A characteristic feature of resummed cross sections, illustrated by this expression, is an integral over scales of the running coupling. Taken literally, these expressions are ill-defined, from regions where the integration variable p_T is of order Λ_{QCD}. Although this singularity can be avoided while retaining NLL accuracy [7], it is also useful to study the implications of such ambiguities, as we shall see below.

The a-dependent expression (5) has yet to be confronted with real LEP data, except for the case $a = 0$, the thrust. Fig. 1 shows data for the closely-related heavy jet mass distribution at the Z pole [10]. The perturbative-only NLL prediction has the right shape overall, but is shifted toward smaller values of jet mass. We will attribute this shift to nonperturbative corrections below.

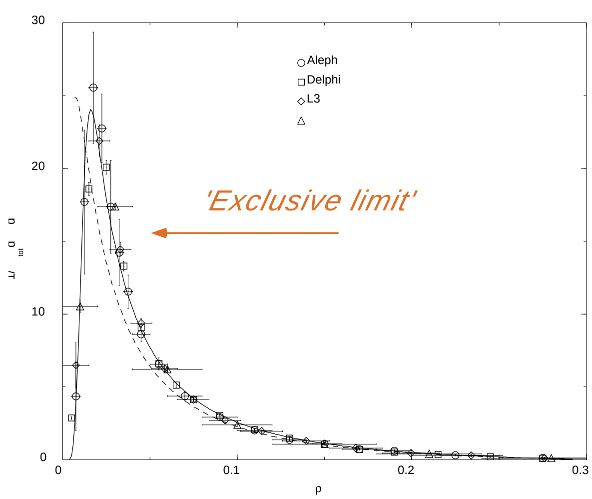

Figure 1: Heavy jet mass distribution at the Z [10]. Dashed line: NLL resummed.

These considerations may be generalized to jet shapes in deeply inelastic lepton-hadron and hadron-hadron scattering when the overall final state can be characterized by a definite set of jets

accompanied by soft radiation, as in Eq. (2) above. Such cross sections, were termed "global" by Dasgupta and Salam in Ref. [11]. Recently, Banfi, Salam and Zanderighi have extended NLL resummation to a wide class of global observables in e^+e^-, DIS and hadron-hadron scattering, by developing an innovative software package [12].

4 Non-global Logs: Color and Energy Flow

Complementary to jet shapes are descriptions of interjet energy flow. A simple illustration is shown in Fig. 2, where we trigger on two jet events in the scattering of particles A and B, and measure the inclusive distribution $\Sigma_\Omega(E)$, where $E \geq E_\Omega \geq 0$, with E_Ω the energy that flows into some angular region Ω, away from both the collision axis and the jet directions. Quantities like $\Sigma_O(E)$ are sometimes referred to as radiators.

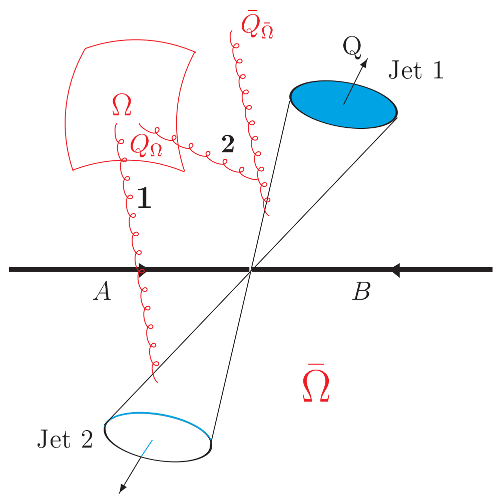

Figure 2: Geometry for energy flow observables.

We can imagine (at least) two choices for such a cross section. First, it may be fully inclusive in the region $\bar{\Omega}$ between Ω and the jets. In this case, the number of jets is not fixed, and the observable is *nonglobal* in the terminology Dasgupta and Salam [11]. This observable cannot be factorized into a fixed number of jets as in Eq. (2), and as such cannot be resummed to a simple exponential in the same way as the event shapes described above. Alternatively, we may limit radiation into region $\bar{\Omega}$ by constructing a correlation with an event shape such as τ_a that fixes the number of jets [8, 13].

Cross sections where the number of jets is not fixed are not fully understood, but they remain infrared safe, so that we should be able to learn about them in perturbation theory. Indeed, Banfi, Marchesini and Smye [14] showed that at leading logarithm, $\alpha_s^n \ln^n(\sqrt{S}/E_\Omega)$, and in the limit of large numbers of colors, N_c, these cross sections obey a beautiful nonlinear evolution

equation,

$$\partial_\Delta \Sigma_{ab}(E) = -\partial_\Delta R_{ab}\, \Sigma_{ab}(E) + \int_{k \text{ not in } \Omega} dN_{ab\to k}\, \left(\Sigma_{ak}(E)\Sigma_{kb}(E) - \Sigma_{ab}(E)\right)\,, \tag{6}$$

where $\partial_\Delta = E\partial_E$. Here $dN_{ab\to k}$ describes the angular distibution of radiation from a pair (dipole) of color charges, and R_{ab} the corresponding logarithmic angular integration, where the β are four-velocities of partons within the jets that radiate soft gluons directly into Ω (case 1 in Fig. 2),

$$dN_{ab\to k} = \frac{d\Omega_k}{4\pi}\, \frac{\beta_a \cdot \beta_b}{\beta_k \cdot \beta_b\, \beta_k \cdot \beta_a} \qquad R_{ab} = \int_E^Q \frac{dE'}{E'}\, \int_\Omega dN_{ab\to k}\,. \tag{7}$$

These quantities control the linear term on the right-hand side of Eq. (6). The nonlinear terms describe radiation from a hard gluon of momentum k into region $\bar{\Omega}$. This gluons acts as a new, recoil-less source of further emission into region Ω (2 in Fig. 2). In the large-N_c limit, any gluon may be thought of as a pair of sources $q(k)\bar{q}(k)$, with quark-antiquark color. In this way, for large N_c, the combination $\bar{q}(a)G(k)q(b)$ is equivalent to an independent pair of dipole sources, $\bar{q}(a)q(k) \oplus \bar{q}(k)q(a)$. An intriguing relation has been pointed out of this equation with small-x evolution in the unitarity (saturation) limit [15].

In the more restrictive approach, the correlation with an event shape, for example an angularity, can fix the number of jets by setting $\tau_a Q \sim E$ [8], and we may factorize and resum as above, $d\sigma/dEd\tau_a \sim S(E/\tau_a Q) \otimes d\sigma_{\rm resum}/d\tau_a$. In a combination of both approaches [13], one may trigger on two narrow jets, and take the limit $E/(\tau_a Q) \to 0$, using the nonlinear evolution above for the function S that describes radiation into the interjet region.

With methods such as these, we can study the influence of color flow at short distances on energy flow at wide angles [16], including applications to final state rapidity gaps [17].

5 From Resummed Perturbation Theory to Nonperturbative QCD

We now return to the interpretation of expressions like (5), where the argument of α_s vanishes in the infrared limit, As required by infrared safety, if we reexpand the running coupling in terms of $\alpha_s(Q)$ for any fixed momentum scale Q, the result is finite at all orders. The divergence associated with the running coupling comes from an $n!$ behavior of this expansion at order α_s^n [18]. We treat such contributions as ambiguities in the perturbative expansion that may be resolved by supplementing the series with nonperturbative parameters, which turn out to be associated with power corrections in the large scale Q. Shape functions [19] organize the dominant corrections for event shapes such as angularities to all powers of Q.

To be specific, in Eq. (5), we treat the region $p_T > \kappa$ using perturbation theory, with $\kappa > \Lambda_{QCD}$ a new factorization scale. For $p_T < \kappa$, we expand the integrand and replace the complete series of powers of $\nu p_T/Q$ by $\tilde{f}_{a,\rm NP}$, the shape function,

$$E(\nu,Q,a) \;=\; E_{\rm PT} + \frac{2}{1-a}\sum_{n=1}^{\infty}\frac{1}{n\,n!}\left(-\frac{\nu}{Q}\right)^n \int_0^{\kappa^2}\frac{dp_T^2}{p_T^2}\, p_T^n\, A\left(\alpha_s(p_T)\right) \equiv E_{\rm PT} + \ln \tilde{f}_{a,\rm NP}\left(\frac{\nu}{Q},\kappa\right) \tag{8}$$

The jet function is now factorized in ν-space into perturbative and nonperturbative functions, resulting in a convolution after the inverse transform (4),

$$\sigma(\tau_a, Q) = \int_0^{\tau_a Q} d\xi \; f_{a,\mathrm{NP}}(\xi) \; \sigma_{\mathrm{PT}}(\tau_a Q - \xi, Q) \,. \tag{9}$$

Because the momentum space shape function is independent of Q, once it is chosen to describe the τ_a-distribution at some fixed Q, say the Z mass, it gives predictions for all Q [9, 19, 20, 21].

Fig. 3 illustrates for the heavy jet mass the quality of fit that may be achieved in this way for a very wide range in Q. What we learn from such an event shape is illustrated by the functional

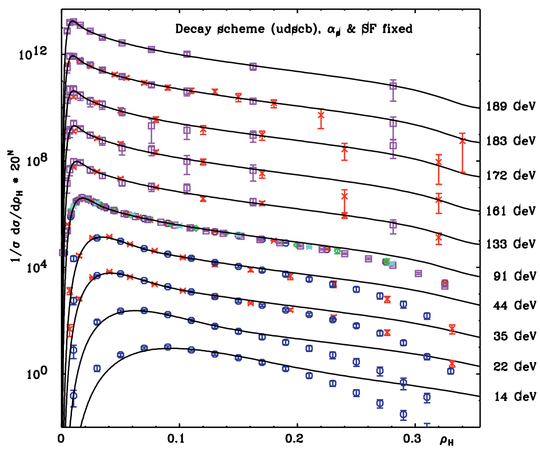

Figure 3: Jet mass fit with a shape function at the Z mass, with predictions and data for other energies.

form discussed in [20] $f_{0,\mathrm{NP}}(\rho) = \mathrm{const}\ \rho^{c-1}\ \mathrm{e}^{-d\rho^2}$. The parameter c may be interpreted in terms of the transverse momenta emitted per unity rapidity range, while d is related to the flow of energy between the hemispheres associated with the jets. More generally, shape functions are related to correlations of energy flow. This connection may be made in a manner that recalls the moment analysis of multiplicity distributions [22]. We introduce an energy flow operator $\mathcal{E}(\Omega)$ by its action on states,

$$\mathcal{E}(\Omega)|k_1, \ldots k_N\rangle = \sum_{j=1}^{N} k_j^0 \delta^2(\Omega - \Omega_j)|k_1, \ldots k_N\rangle \,. \tag{10}$$

For $\tau_a \to 0$, the final state is characterized by two narrow jets accompanied by soft radiation. In this limit, moments of the shape function may be represented as [19]

$$\int_0^{\infty} d\xi \, \xi^n f_a(\xi) = \int \prod_{j=1}^{n} d\Omega_i \; w_a(\cos\theta_j) \langle \mathcal{E}(\Omega_1) \ldots \mathcal{E}(\Omega_n) \rangle \,, \tag{11}$$

where the angular functions $w_a(\cos\theta)$, have been defined above in Eq. (3). The expectations in (11) are taken in the presence recoilless color sources (Wilson lines) that accurately represent the

coupling of the soft radiation to the jets. The logarithm of the shape functions is a cumulant expansion in these expectations [20].

By generalizing the thrust and related event shapes to angularities, we recognize in Eq. (8) an interesting scaling property for the associated event shapes event shapes, which may be thought of as a test of the rapidity-independence of nonperturbative dynamics [9, 21],

$$\tilde{f}_a\left(\frac{\nu}{Q},\kappa\right) = \left[\tilde{f}_0\left(\frac{\nu}{Q},\kappa\right)\right]^{\frac{1}{1-a}} . \tag{12}$$

It would be interesting to confront this prediciton with LEP data. For the present, however, we must content ourselves with a comparison to PYTHIA, which is fairly encouraging, as Fig. 4 shows [21]. It is worth emphasizing that most event shapes were invented for the e$^+$e$^-$ jet physics

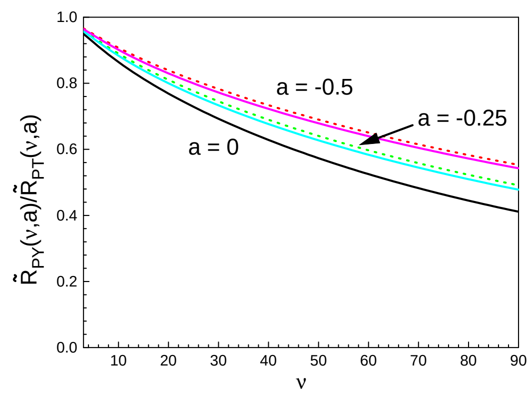

Figure 4: Scaling predictions for the shape function in a as tested by PYTHIA.

of the late 70's, and there is more to learn by addressing existing data with new analyses [23].

Also relatively unexplored are power corrections for single particle inclusive cross sections. Here we would like to understand single-particle cross sections from fixed target to collider energies. Using the formalism of Ref. [24], and the analysis of resummed cross sections described above, it is possible to study the energy dependence of power corrections in cross sections at measured $x_T = 2p_T/\sqrt{S}$ [25]. For example, in the phase space limit, $x_T \to 1$ for $A+B \to \gamma+X$, the cross section may be written as the inverse transform of the Mellin moments of the fixed-order cross section with a resummation of higher order logarithms, in a form analogous to (5),

$$p_T^3\,\frac{d\sigma_{AB\to\gamma}}{dp_T} \quad\sim\quad \int_{-i\infty}^{i\infty}\frac{dN}{2\pi i}\,\tilde{\sigma}^{(0)}_{AB\to\gamma}(N)\ \left(x_T^2\right)^{-N-1}\ \mathrm{e}^{E_{\rm PT}(N,p_T)}\ \mathrm{e}^{\delta\,E_{\rm NP}(N,p_T)} . \tag{13}$$

Isolating low scales as above, we derive a nonperturbative exponent with the N dependence

$$\delta\,E_{\rm NP}(N) = \text{const.}\,\frac{N^2}{p_T^2}\ \ln\frac{p_T}{N} \quad\Rightarrow\quad \delta\,E_{NP}(x_T) = \text{const.}\,\frac{1}{p_T^2\ln^2\left(\frac{4p_T^2}{S}\right)}\ \ln\left(p_T\ln\left(\frac{4p_T^2}{S}\right)\right) , \tag{14}$$

where we have used the conjugate relation of the variables N and $\ln x_T^2$. We find a complex behavior in S at fixed p_T, associated with energy conservation [25].

6 Hopeful Conclusion

Resummations can bring perturbative QCD to the doorstep of nonperturbative field theory. This analysis is still evolving, and alternative treatments of the perturbative/nonperturbative transition are possible. Theoretical and experimental studies of the interplay of color and energy flow in hadronic scattering should aid these developments. Eventually we will learn to translate fully the language of partons into the language of hadrons.

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